Lecture 27 Compare Two Populations - Proportion

BIO210 Biostatistics

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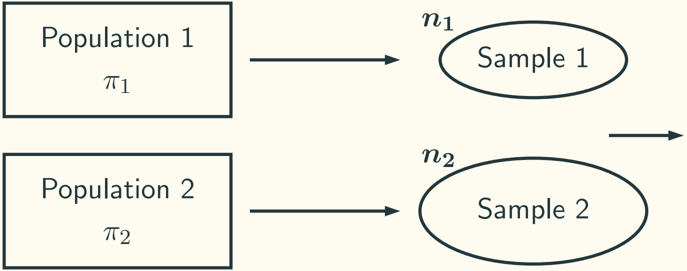
Compare two proportions

Whether the proportions of colour blindness are the same in two different populations (e.g. male vs female, Asian vs European) ?

Whether chemical A is better than chemical B for culturing cells in petri dishes (can be measured by percentage of cells that express *Pou5f1*) ?

Whether drug A is more efficient than drug B in terms of curing a certain disease (can be measured by percentage of cured patients) ?



$$H_0: \quad \pi_1 = \pi_2, \ \pi_1 \geqslant \pi_2, \ \pi_1 \leqslant \pi_2$$

$$H_1: \quad \pi_1 \neq \pi_2, \ \pi_1 < \pi_2, \ \pi_1 > \pi_2$$

ABO Blood Types And The COVID-19

Clinical Infectious Diseases

BRIEF REPORT

Relationship Between the ABO Blood Group and the Coronavirus Disease 2019 (COVID-19) Susceptibility

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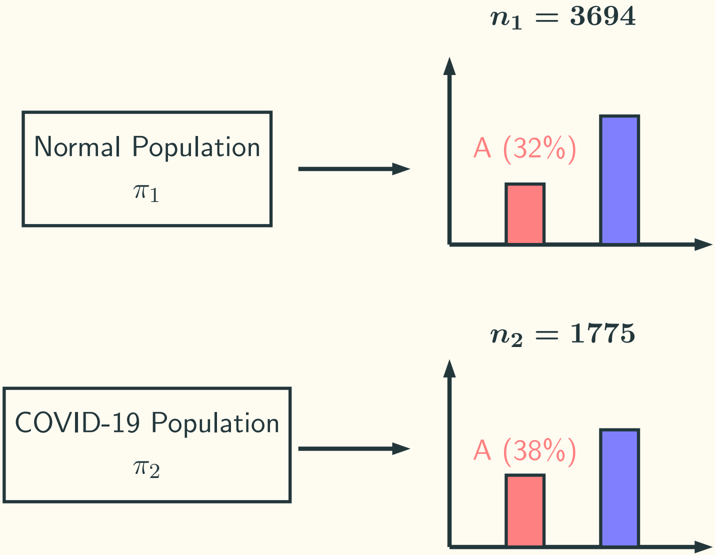
Type A blood in normal people and COVID-19 patients



Is π_1 different from π_2 ?

$$H_0: \ \pi_1 = \pi_2$$

$$H_1: \ \pi_1 \neq \pi_2$$

Strategy 1: Use One-sample Hypothesis Testing ??

Two choices:

- $H_0: \ \pi_1 = 0.38$

  $H_1: \ \pi_1 \neq 0.38$
- $H_0: \ \pi_2 = 0.32$

  $H_1: \ \pi_2 \neq 0.32$

Two anwsers:

- $z = -7.5$

  $p = 6.4 \times 10^{-14}$
- $z = 4.4$

  $p = 1.1 \times 10^{-5}$

Strategy 2: Figure Out The Sampling Distribution of The Difference

- Let the random variable $\boldsymbol{P_1}$ represnt the proportion of blood type A in a sample $(n_1 = 3694)$ drawn from normal people.
- Let the random variable $\boldsymbol{P_2}$ represnt the proportion of blood type A in a sample $(n_2 = 1775)$ drawn from COVID-19 patients.

Normal
π_1

COVID-19
π_2

$$\boldsymbol{P}_1 \sim \mathcal{N}\left(\mu_{\boldsymbol{P}} = \pi_1,\ \sigma^2_{\boldsymbol{P}} = \frac{\pi_1(1-\pi_1)}{n_1}\right)$$

$$\boldsymbol{P}_2 \sim \mathcal{N}\left(\mu_{\boldsymbol{P}} = \pi_2,\ \sigma^2_{\boldsymbol{P}} = \frac{\pi_2(1-\pi_2)}{n_2}\right)$$

$$\boldsymbol{\delta} = \boldsymbol{\pi_1} - \boldsymbol{\pi_2}$$

$$\boldsymbol{D} = \boldsymbol{P}_1 - \boldsymbol{P}_2$$

$$\boldsymbol{D} \sim \boldsymbol{?}$$

Sampling Distribution of The Difference of The Sample Proportion

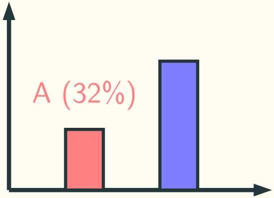
- $\boldsymbol{D} \sim \mathcal{N}\left(\pi_1 - \pi_2,\, \dfrac{\pi_1(1-\pi_1)}{n_1} + \dfrac{\pi_2(1-\pi_2)}{n_2}\right)$
- $\boldsymbol{D} = \boldsymbol{P}_1 - \boldsymbol{P}_2$ and $d = p_1 - p_2$ are the point estimator/estimate of δ
- 95% CI: $(p_1 - p_2) \pm 1.96\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$

Type A blood in normal people and COVID-19 patients

Normal Population

π_1

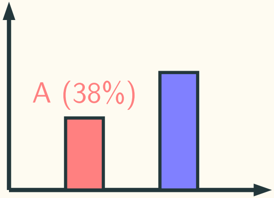
$\boldsymbol{n_1 = 3694}$



COVID-19 Population

π_2

$\boldsymbol{n_2 = 1775}$



Is π_1 different from π_2 ?

$$H_0 : \ \pi_1 = \pi_2$$

$$H_1 : \ \pi_1 \neq \pi_2$$

$$\downarrow$$

$$H_0 : \ \delta = \pi_1 - \pi_2 = 0$$

$$H_1 : \ \delta = \pi_1 - \pi_2 \neq 0$$

Two-sample Hypothesis Testing For Proportion

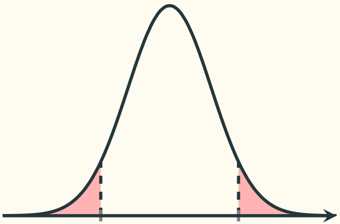
$$H_0:\ \delta = \pi_1 - \pi_2 = 0$$

$$H_1:\ \delta = \pi_1 - \pi_2 \neq 0$$

$$\boldsymbol{D} \sim \mathcal{N}\left(\pi_1 - \pi_2,\ \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}\right) \xrightarrow[\text{were true}]{\text{if } H_0} \boldsymbol{D} \sim \mathcal{N}\left(0, \left(\frac{1}{n_1} + \frac{1}{n_2}\right)\pi(1-\pi)\right)$$

1. What we observe is: $d = p_1 - p_2$
2. What is the probability of observing d or more extreme?

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\pi(1-\pi)}} = \frac{p_1 - p_2}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\pi(1-\pi)}}$$



What is the best estimate for π?

Two-sample Hypothesis Testing For Proportion

	Normal	COVID-19
A	a	b
Non-A	c	d
Total	n_1	n_2

Sample size: bigger is always better:

$$\pi : \frac{a+b}{n_1+n_2} = \frac{n_1 p_1 + n_2 p_2}{n_1+n_2} = p$$

The test statistic:

$$z = \frac{p_1 - p_2}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) p(1-p)}}$$

The test statistic:

$$p = \frac{1188+670}{3694+1775} = 0.34,\ z = \frac{0.32 - 0.38}{\sqrt{\left(\frac{1}{3694} + \frac{1}{1775}\right) \times 0.34 \times 0.66}} = -4.4$$

Example: Two-sample Hypothesis Testing For Proportion

Myopia: Researchers suspect that myopia, or nearsightedness, is becoming more common over time. A study from the year 2000 showed 139 cases of myopia in 420 randomly selected people. A separate study from 2015 showed 228 cases in 600 randomly selected people. Perform a hypothesis testing to see if the researchers' suspicion is true or not.

Sample statistics: $n_1 = 420$, $p_1 = \dfrac{139}{420} = 0.33$, $n_2 = 600$, $p_2 = \dfrac{228}{600} = 0.38$

Pooled estimate for π: $p = \dfrac{139 + 228}{420 + 600} = 0.36$

The test statistics: $z = \dfrac{p_1 - p_2}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) p(1-p)}} = \dfrac{0.33 - 0.38}{\sqrt{\left(\frac{1}{420} + \frac{1}{600}\right) \times 0.36 \times 0.64}}$