Lecture 28 Compare Two Populations - Mean

BIO210 Biostatistics

Xi Chen

Fall, 2025

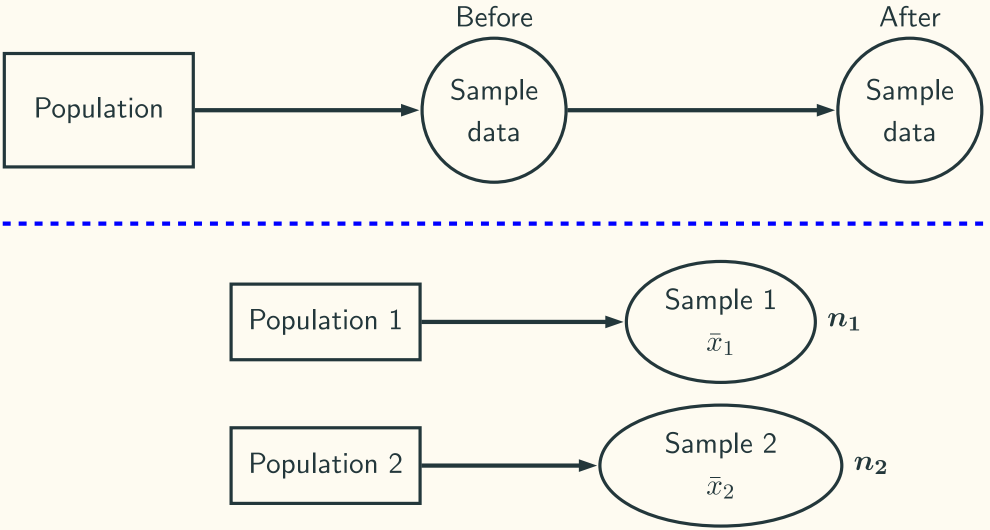
School of Life Sciences
Southern University of Science and Technology

南方科技大学生命科学学院
SUSTech · SCHOOL OF
LIFE SCIENCES

Compare Two Means

Paired samples	Independent samples	
Whether the body temperatures rise after exercise ? Whether the body temperatures are different comparing morning to evening ?	Whether chemical A is more efficient than chemical B in terms of promoting the production of protein C in bacteria ?	Whether the blood pressures of COVID-19 patients are the same as normal people ?

Scenarios of Comparing Two Means



Scenarios of Comparing Two Means - Paired Sample Design

- **Hypertension:** A new vitamin tablet is used as dietary supplement to keep people healthy. Supposed we are concerned about its effect on the blood pressure of people.

- **Design 1:** Longitudinal study
  1. Choose a random group of people who have not taken the tablet, measure their blood pressure as the baseline blood pressure.
  2. Revisit those people after 1 year to ascertain a subgroup of people who have become the tablet user. This is our study population.
  3. Measure the blood pressure of the study population at the follow-up visit.
  4. Compare baseline blood pressure and the follow-up blood pressure.

Scenarios of Comparing Two Means - Independent Samples Design

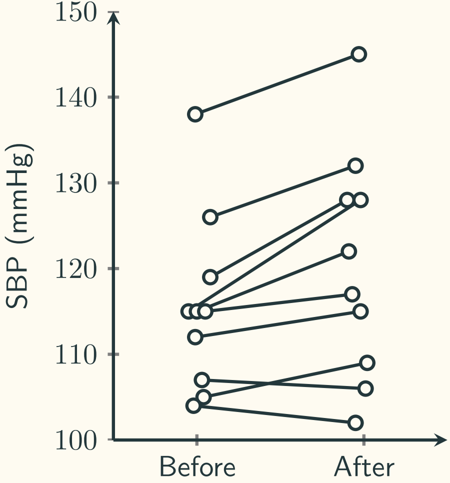
- **Hypertension:** A new vitamin tablet is used as dietary supplement to keep people healthy. Supposed we are concerned about its effect on the blood pressure of people.

- **Design 2:** Cross-Sectional Study
  1. Identify both a group of the tablet users and a group of non-tablet users.
  2. Measure the blood pressure of both groups and compare them.

Comparing Two Means - Longitudinal study

The systolic blood pressure (SBP) levels (mm Hg) of 10 people before and after taking the tablet:

Person	Before	After	Difference
1	115	128	13
2	112	115	3
3	107	106	-1
4	119	128	9
5	115	122	7
6	138	145	7
7	126	132	6
8	105	109	4
9	104	102	-2
10	115	117	2



Comparing Two Means - Longitudinal study

Let δ be the true difference of SBP before and after taking the tablet.

		Difference
D_1	d_1	13
D_2	d_2	3
D_3	d_3	-1
D_4	d_4	9
D_5	d_5	7
D_6	d_6	7
D_7	d_7	6
D_8	d_8	4
D_9	d_9	-2
D_{10}	d_{10}	2

- Does the daily intake of the tablet change the blood pressure of people ?

H_0 : it does not, $\delta = 0$

H_1 : it does, $\delta \neq 0$

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i \sim ? \qquad \bar{D} \sim \mathcal{N}\left(\delta, \frac{\sigma_d^2}{n}\right)$$

95% CI : $\bar{d} \pm Z_{0.025}\frac{\sigma_d}{\sqrt{n}}$ or $\bar{d} \pm t_{0.025,n-1}\frac{s_d}{\sqrt{n}}$

Comparing Two Means - Longitudinal study

Hypothesis testing:

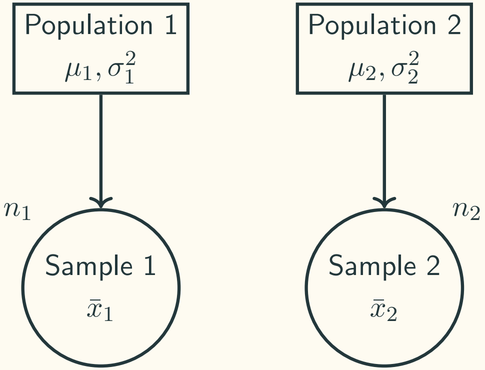
We know that: $\bar{D} \sim \mathcal{N}\left(\mu = \delta, \sigma^2 = \frac{\sigma_d^2}{n}\right)$

If the null hypothesis were true: $\bar{D} \sim \mathcal{N}\left(\mu = 0, \sigma^2 = \frac{\sigma_d^2}{n}\right)$

The test statistics: $z = \frac{\bar{d}}{\sigma_d/\sqrt{n}}$ or $t = \frac{\bar{d}}{s_d/\sqrt{n}}$

This example: $t = \frac{4.8}{4.566/\sqrt{10}} = 3.32$

Comparing Two Means - Independent Samples



$$\bar{\boldsymbol{X}}_{\mathbf{1}} \sim \mathcal{N}\left(\mu_1, \frac{\sigma_1^2}{n_1}\right) \qquad \bar{\boldsymbol{X}}_{\mathbf{2}} \sim \mathcal{N}\left(\mu_2, \frac{\sigma_2^2}{n_2}\right)$$

Is μ_1 equal to μ_2 ?

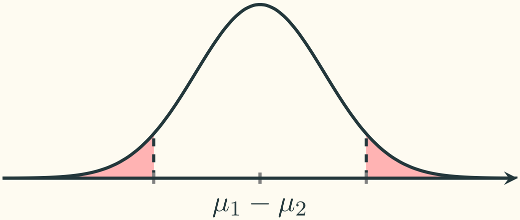
$$\begin{cases} H_0 : & \mu_1 = \mu_2 \\ H_1 : & \mu_1 \neq \mu_2 \end{cases}$$

$$\Leftrightarrow \begin{cases} H_0 : & \delta = \mu_1 - \mu_2 = 0 \\ H_1 : & \delta = \mu_1 - \mu_2 \neq 0 \end{cases}$$

$$\boldsymbol{D} = \bar{\boldsymbol{X}}_{\mathbf{1}} - \bar{\boldsymbol{X}}_{\mathbf{2}} \sim ?$$

Sampling Distribution of The Difference of The Sample Means

$$\boldsymbol{D} \sim \mathcal{N}\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$



95% CI: $(\bar{x}_1 - \bar{x}_2) \pm 1.96\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

?

Scenario 1: Equal Variance - Student's t-test

When $\sigma_1^2 = \sigma_2^2 = \sigma^2$:

$$\boldsymbol{D} \sim \mathcal{N}\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right) \Rightarrow \boldsymbol{D} \sim \mathcal{N}\left(\mu_1 - \mu_2, \sigma^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]\right)$$

Pooled estimate (weighted average of the sample variances) for the common variance σ^2 :

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}, \text{where } s_1^2 = \frac{\sum_{i=1}^{n_1}(x - \bar{x}_1)^2}{n_1 - 1} \text{ and } s_2^2 = \frac{\sum_{i=1}^{n_2}(x - \bar{x}_2)^2}{n_2 - 1}$$

$$\boldsymbol{t} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim \boldsymbol{t_{n_1+n_2-2}}$$

If H_0 were true, we could use the test statistic:

$$\boldsymbol{t} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Scenario 2: Unequal Variance - Welch's t-test

When $\sigma_1^2 \neq \sigma_2^2$:

$$\boldsymbol{t} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim \boldsymbol{t_\nu}\text{, where } \nu \approx \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

If H_0 were true, the test statistic:

$$\boldsymbol{t} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, \nu \approx \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

$\leftarrow$ Welch-Satterthwaite approximation

Why Bother And How To Choose?

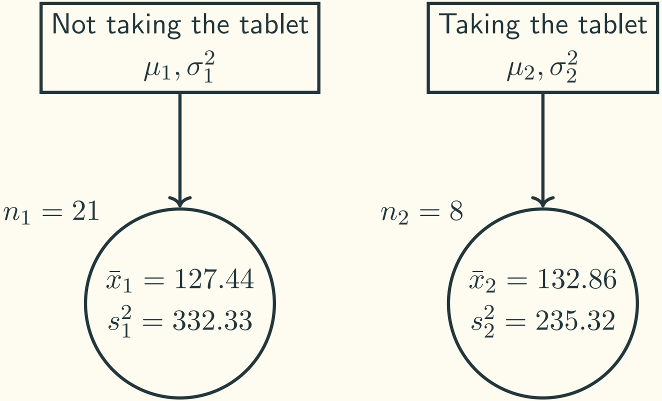
- Why Bother ?

  - The basis of hypothesis testing: if H_0 were true, then the test statistic should $\sim$ sampling distribution.

- How To Choose ?

  - Different opinions. Note: t-tests are robust

Hypertention: A new vitamin tablet is used as dietary supplement to keep people healthy. Supposed we are concerned about its effect on the blood pressure of people.



What can be said about the underlying mean difference in blood pressure between the two groups?